PDE-DRIVEN LEVEL SETS, VELOCITY FIELDS AND PERIMETER PENALIZATION FOR STRUCTURAL TOPOLOGY OPTIMIZATION

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The general problem of topology optimization of a structure is an *ill-posed* problem in its mathematical theory and numerical methods [1,2]. Homogenization methods have been extensively developed in recent years and evolved into the state-of-the-art [1]. The technique is known as *relaxation* and, as a result, solutions to the relaxed problem are guaranteed to exist, although the optimal solutions commonly have perforated microstructures in the resulting design, as expected in consistence with the relaxation [2]. Therefore, it becomes necessary to be able to suppress perforated microstructures in the optimal design by modifying the relaxed formulation. Several suppression techniques have been developed. Introducing *a priori* restrictions on the configuration of the microstructure is one approach [1,2], while the suppression may also be achieved by explicitly penalizing intermediate values of the bulk density. The later technique becomes quite popular for its conceptual and practical simplicity [2]. Although these "*engineering approaches"* of suppression have been widely applied to problems with multiple physics and multiple materials, various fundamental issues remain as obstacles [2,3]. The suppressions do not directly address the chattering problem underlying the relaxation concept.

This paper addresses the problem of structural shape and topology optimization. A level set method [3] is adopted as an alternative approach to the popular homogenization based methods. The paper focuses on four areas of discussion: *(1)* The level-set model of the structure's shape is characterized as a region and global representation; the shape boundary is embedded in a higher-dimensional scalar function as its "iso-surface." Changes of the shape and topology are governed by a partial differential equation (PDE). *(2)* The velocity vector of the Hamilton-Jacobi PDE is shown to be naturally related to the shape derivative from the classical shape variational analysis. Thus, the level set method provides a natural setting to combine the rigorous shape variations into the optimization process. *(3)* Perimeter regularization is incorporated in the method to make the optimization problem well-posed. It also produces an effect of the geometric heat equation, regularizing and smoothing the geometric boundaries as an anisotropic filter. *(4)* We further describe numerical techniques for efficient and robust implementation of the method, by embedding a rectilinear grid in a fixed finite element mesh defined on a reference design domain. This would separate the issues of accuracy in numerical calculations of the physical equation and in the level-set model propagation. Finally, the benefit and the advantages of the developed method are illustrated with several 2D examples that have been extensively used in the recent literature of topology optimization, especially in the homogenization based methods.

References

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